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Bonabeau model on a fully connected graph

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Abstract. Numerical simulations are reported on the Bonabeau model on a fully connected graph, where spatial degrees of freedom are absent. The control parameter is the memory factor f. The phase transition is observed at the dispersion of the agents power h_i. The critical value f_C shows a hysteretic behavior with respect to the initial distribution of h_i. f_C decreases with the system size; this decrease can be compensated by a greater number of fights between a global reduction of the distribution width of h_i. The latter step is equivalent to a partial forgetting.

PACS. 89.20.-a Interdisciplinary applications of physics – 89.65.Ef Social organizations; anthropology – 87.23.Ge Dynamics of social systems

1 Introduction

The game theory is considered to be mathematical formulation of theory of conflicts [1]. Since its initialization by von Neumann and Morgenstern in 1946 [2], it becomes a powerful branch of knowledge, with numerous links to other areas of science. Offering new insight into old problems, these links allow to widen our perspective to new interdisciplinary applications.

Such is the concept of self-organizing hierarchies in animal societies, introduced by Bonabeau et al. in 1995 [3] (see Refs. [4–6] for the most recent results and Ref. [7] for reviews). In the Bonabeau model, N agents walk randomly on a given area and fight when they met. Initially, the outcome of fights is random. However, agents are able to remember for some time their past results and this memory influences their subsequent fights. Then, there are two competitive mechanisms. First, each fight influences the agents' power: winner is stronger and looser is weaker, what alters the probabilities of winning of their future fights. Second, the information of these alterations is gradually erased. As a consequence, a phase transition can be observed: for a given velocity of forgetting, frequent fights produce a hierarchy of permanent winners and permanent loosers. This hierarchy is maintained in time. However, if fights are rare, the hierarchy is being forgotten quicker, than it is reproduced. The frequency of fights depends on the number of fighters on a given area. The order parameter is the dispersion of power of the agents, or the dispersion of probabilities of winning/loosing of pairs of agents. The phase transition was termed in [5] as the one between hierarchical and egalitarian society. Besides its sociological associations, the Bonabeau model offers nontrivial dynamics of a specific, time-dependent spontaneous symmetry breaking, which — in our opinion — deserves attention from a theoretical point of view. Unfortunately, numerical experiments described in reference [5] have shown that the order and even the appearance of the phase transition depends on a direct way of forgetting, i.e. the time dependence of the agents' power, and on the presence of mean-field-like coupling which is absent in reference [3].

A reasonable strategy of resolving this puzzle seems to separate it to elements as simple as possible, and to observe their properties. Here we propose a formulation where the spatial coordinates of agents are absent. Instead, the agents are placed at nodes of a fully connected graph, i.e. each agent can meet every other agent. Scale free networks were mentioned in [6]. In next section the applied procedure is described in detail. Numerical results are reported and concluded in two subsequent sections.

2 The model

Two fighters i and j are selected randomly from a population of N agents. The probability that i-th agent wins

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over j-th is

$$P(i,j,t) = \frac{1}{1+\exp\{\sigma(t)[h_j(t)-h_i(t)]\}} \quad (1)$$

where h_i is the power of i-th agent at time t and

$$\sigma^2(t) = \langle h_i^2(t)\rangle - \langle h_i(t)\rangle^2, \quad (2)$$

where $\langle\cdots\rangle$ is the average over N agents. As an output of the fight, the power h_i of the winner increases by ε and the one of the looser decreases by ε.

Every N_f steps (i.e. fights), the powers h_i of all agents are multiplied by the factor $(1-f)$, where $f \in (0,1)$. This is the step of 'forgetting'. As often as forgetting procedure takes place the current value of dispersion σ is evaluated which is then fixed during next N_f fights. The number of such updates of σ is N_{iter}. Then, the total number of fights during one simulation is $N_f N_{iter}$.

3 Results

The parameters of the calculations are: the system size N, the number of fights N_f between the subsequent updates of σ, the number of steps N_{iter} and the change ε of the power h_i. The initial distribution of h_i appears also to be relevant. This is set either random ($h_i \in [-N/2, N/2]$), or homogeneous, i.e. $h_i = i\varepsilon$ for all i, or delta-like, i.e. $h_i = 0$ for all i. We keep $\varepsilon = 0.01$. The output of the simulation is the critical value of f, i.e. f_C, where σ changes abruptly. We can speak about 'hierarchical' (large σ) or 'egalitarian' (small σ) society. As a rule, for small f we get hierarchy, and for large f — equality.

It appears that f_C depends on the ratio between N and N_f. Keeping N_f constant and increasing N, we make forgetting more and more relevant, because each agent fights less between subsequent forgettings. Then, the critical value f_C decreases with N, as shown in Figure 1a. We can compensate this variation, changing N_f and N simultaneously as to keep f_C constant. Two series of this procedure are shown in Figures 1b and 1c, for $f_C = 0.3$ and $f_C = 0.5$. In Figure 2, we show two respective curves of N_f against N.

We note that the obtained values of σ are stable vs time, because the distribution of the agent's power h_i stabilize after some transient time. Examples of this dynamics are shown in Figure 3 for various values of the parameters.

According to what was said above, the system size of the critical value of forgetting parameter $f_C(N)$ decreases to zero for large N. The character of this variation is shown in Figure 4. It is likely that there is a power-like behavior, i.e. $f_C \propto N^{-\alpha}$, with $\alpha \approx 0.88$.

For small systems, the final value of σ does not depend on the initial distribution of h_i. However, above $N = 175$ the final distribution of h_i shows some hysteretic behavior, i.e. σ does depend on initial values of h_i. In Figure 5 we show the curves $\sigma(f)$ obtained for random, homogeneous or delta-like initial values of h_i. The curves, identical for

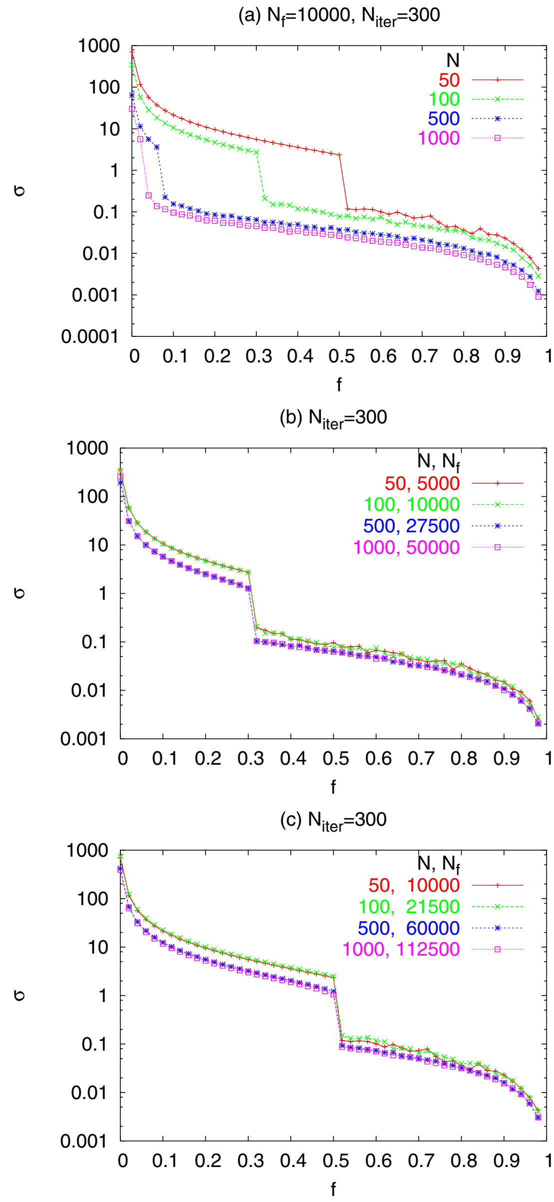

Fig. 1. Dependence of $\sigma(f)$ for (a) fixed $N_f = 10^4$ and different values of N and with tuned values of N_f for N-independent values of f_C: (b) $f_C = 0.3$, (c) $f_C = 0.5$.

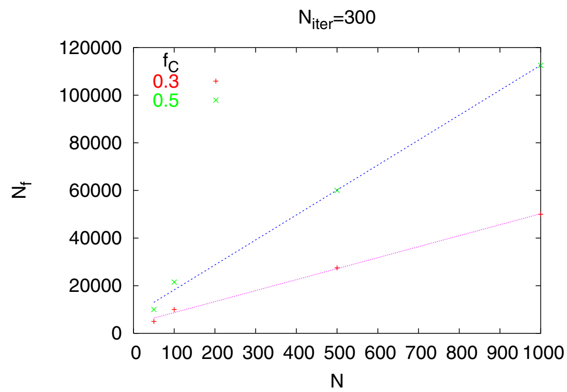

Fig. 2. Dependence of $N_f(N)$ for different values of f_C. Linear fits are $N_f = 105N + 7756$ ($f_C = 0.5$) and $N_f = 46N + 4095$ ($f_C = 0.3$).

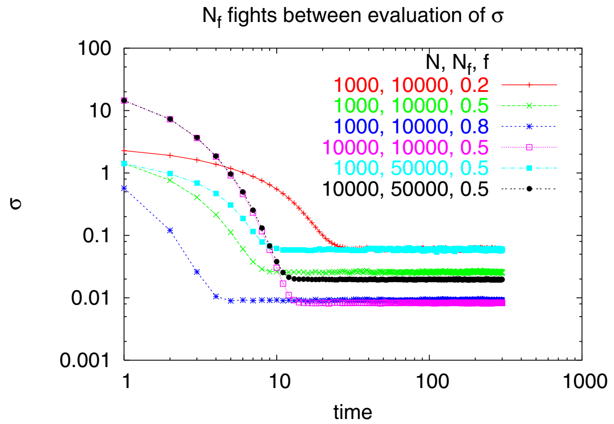

Fig. 3. Time evolution of σ. Between subsequent σ evaluations N_f fights take place.

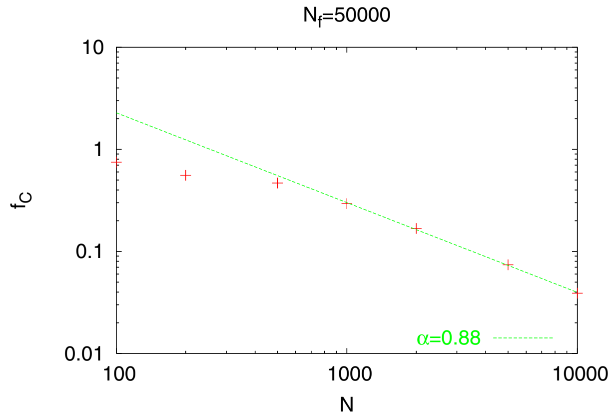

Fig. 4. Dependence of the critical value of the forgetting parameter f_C on the system size N for $N_f = 50\,000$. The least-square fit $f_C(N) \propto N^{-0.88}$ to the last decade is included.

$N < 175$, split abruptly for larger systems. On the contrary, we checked that the hysteretic effect is not observed in the approach in reference [5], at least near the critical point reported there.

4 Discussion

In their seminal paper [3], the authors put several examples and suggestions on societies, which could self-organize into the hierarchical state. There, reference are given on animal societies, like bees, wasps, ants, chickens, cows and ponies. A question arises how important are spatial degrees of freedom in these societies. This is a central task for our considerations, because the territorial aspect of the problem of hierarchy is disregarded here, while it was included in earlier work. It seems that in small groups of some animals, the difference in hierarchy is not to be here or there but rather to do this or that. This is particularly true in societies of some primates, as *Pan troglodytes*, *Homo sapiens* or *Gorilla gorilla* [8–10].

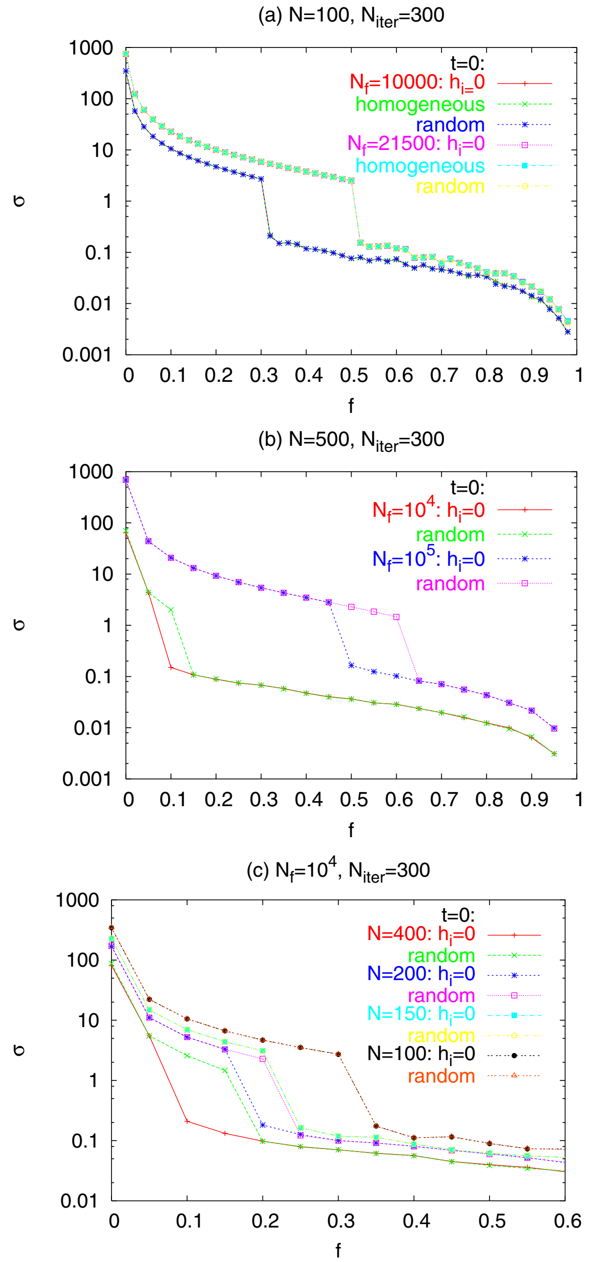

Fig. 5. (a) Final values of σ in small societies ($N = 100$) do not depend on an initial distribution of h_i. Different values of N_f do not alter this result but shift the critical value f_C. (b) When system is large enough ($N = 500$), the critical point f_C for given number of fights N_f depends on the initial conditions. (c) The initial inequality influences the critical point for $N \geq 175$.

The Bonabeau model, once released, lives with its own life as it provides nontrivial questions on its mathematical properties. However, its core is the sociological application: description of a group of agents, possibly human, which concentrate on their hierarchy. Such hierarchy appears in a natural way in sports competitions. For example, each football league in the world produces one table at the end of the season with the numbers of points and goals for each team [12]. Usually, the extremes at the top and the bottom have widely spaces points, while in the center the average teams differ just by one point. This is not dissimilar to the power distribution of the Bonabeau model [6].

How important is the hierarchy for the agents, depends on their experience and social environment. Except army and some universities, we are interested in formation of a society rather egalitarian than hierarchical, and tasks of educational organizations following this attitude are well known [11]. In the Bonabeau model, these efforts get a well-defined purpose: to be on the right side of the transition.

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